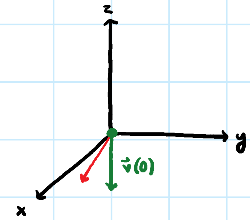
Integrals & Trajectories Cont.

Monday, May 22, 2023 9:33 AM

ex 2) particle starts @ $\vec{r}(0) = \langle 0, 0, 0 \rangle$ with velocity $\vec{v}(0) = \langle 0, 0, -4 \rangle$. mass is $m = 5$ kg & force applied to it is $\vec{F}(t) = \langle \cos^2(t), 0, -t \rangle$, then find $\vec{r}(10)$.



solution: since $\vec{F}(t) = m \cdot \vec{a}(t)$, then $\vec{a}(t) = \frac{\vec{F}(t)}{m} = \left\langle \frac{\cos^2(t)}{5}, 0, \frac{-t}{5} \right\rangle = r(t)'' = v(t)'$

(newton's 2nd law)

thus $\vec{v}(t) = \int_0^t \vec{a}(t) + \vec{v}(0)$ → $\langle 0, 0, -4 \rangle$

$$= \left\langle \frac{1}{5}\left(\frac{\sin(2t)}{4} + \frac{t}{2}\right), 0, \frac{-t^2}{10} - 4 \right\rangle$$

$$\vec{v}(t) = \left\langle \frac{\sin(2t)}{20} + \frac{t}{10}, 0, -\frac{t^2}{10} - 4 \right\rangle$$

* equation of $\vec{v}(t)$ (or $\int \vec{a}(t)$) needs to start @ initial point ($\vec{v}(0)$) if definite integral starting @ $t = 0$ / = constant *

integration of $\frac{\cos^2(t)}{5}$:

$$\cos^2(t) = \frac{1 + \cos(2t)}{2}$$

$$\int_0^t \frac{\cos^2(t)}{5} = \frac{1}{5}\int_0^t \cos^2(t) = \frac{1}{5}\int_0^t \frac{1 + \cos(2t)}{2}$$

$$= \frac{1}{5}\int_0^t \frac{1}{2} + \frac{\cos(2t)}{2} = \frac{1}{5}\left[\frac{t}{2} + \frac{\sin(2t)}{4}\right]_0^t$$

$$= \frac{1}{5}\left(\frac{t}{2} + \frac{\sin(2t)}{4} - \frac{0}{2} + \frac{\sin(0)}{4}\right)$$

$$= \frac{1}{5}\left(\frac{\sin(2t)}{4} + \frac{t}{2}\right)$$

add in after integration

for $\vec{r}(t) = \int_0^t \vec{v}(t) + \vec{r}(0)$

$$= \int_0^t \left\langle \frac{1}{20}\sin(2t) + \frac{t}{10}, 0, -\frac{t^2}{10} - 4 \right\rangle + \langle 0, 0, 0 \rangle$$

$$= \left\langle -\frac{1}{40}\cos(2t) + \frac{t^2}{20} - \left(-\frac{1}{40}\cos(0) + \cancel{\frac{0^2}{20}}\right), 0, -\frac{t^3}{30} - 4t - \left(\cancel{\frac{0^3}{30}} - \cancel{4(0)}\right) \right\rangle$$

$$\vec{r}(10) = \left\langle -\frac{1}{40}\cos(20) + \frac{100}{20} + \frac{1}{40}, 0, -\frac{1000}{30} - 40 \right\rangle$$

$$\vec{r}(10) = \left\langle -\frac{1}{40}\cos(20) + \frac{201}{40}, 0, -\frac{100}{3} - 40 \right\rangle$$

y-coordinate stay zero